PMR-Reduction for Continuous Time OFDM Transmit Signals by Selected Mapping

Jakob Rettelbach* and Johannes B. Huber*
*Institute for Information Transmission, Universität Erlangen-Nürnberg, Erlangen, Germany
Email: {rettelbach,huber}@lnt.de

***Abstract*—In order to cope with the problems of multipath propagation in communication systems, OFDM is currently the most popular choice amongst digital modulation schemes. The main drawback of OFDM is the high peak-to-average power ratio (PAPR), which PAPR necessitates a highly linear high power amplifier (HPA) in Class-A mode to avoid out-of-band radiation. This kind of amplifiers usually suffers from a bad power efficiency. A recent approach is to deploy different amplifier architectures with potentially higher power efficiencies, such as polar amplifiers. These kinds of amplifiers necessitate signals with low peak-to-minimum power ratio (PMR). In this paper, the PMR of the continuous time transmit signal is analyzed and reduced via Selected Mapping (SLM). To achieve this reduction in the domain of digital signal processing an approximation method for the PMR is derived.**

I. Introduction

In digital communications, OFDM is currently the modulation scheme that is mostly deployed to conquer the challenges imposed by multipath propagation. Unique-Word OFDM is a new OFDM alternative that exhibits some advantages compared to classical Cyclic-Prefix OFDM. The main drawback of OFDM is a high peak-to-average power ratio which necessitates a highly linear amplifier with bad power efficiency. In recent approaches, different amplifier structures like polar amplifiers (cf., e.g., [1], [2]) are being tested to amplify OFDM signals with higher efficiency.

Polar amplifiers are structured as shown in the block diagram of Fig. 1. A polar amplifier basically acts as a digital-to-RF converter: Firstly, the signal is converted from usual I-/Q-components represented in the equivalent complex baseband domain (ECB-domain) to amplitude and phase representation, i.e. transformed into polar coordinates. The carrier wave at carrier frequency is phase modulated by the phase signal. This RF-signal is fed into an RF-amplifier where the envelope/amplitude signal acts like a supply voltage. At the output of this amplifier the desired amplified amplitude- and phase-modulated RF-signal is obtained. When the transmit signal passes closely or even crosses the origin of the complex plane, rapid phase changes can be observed. These discontinuities in the phase signal lead to high bandwidth requirements. Therefore, signal values with small magnitude should be eliminated to avoid these discontinuities. Together with the similarly negative property of high signal peaks, a transmit signal is appropriately characterized by the peak-to-minimum power ratio (PMR) [3]. The PMR of a continuous time transmit signal $\mathrm{s}(t)$ (ECB-domain) is defined as:

$$\mathrm{PMR} = \frac{\max_t |\mathrm{s}(t)|^2}{\min_t |\mathrm{s}(t)|^2}. \quad (1)$$

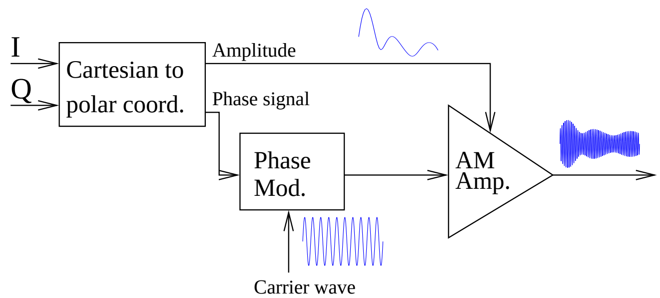

Fig. 1. Block diagram: Polar amplifier structure

In [4], Selected Mapping (SLM) was used to reduce the PMR for Cyclic-Prefix (CP-) and Unique-Word (UW-) OFDM for T-spaced signaling, but the continuous time transmit signal is of more interest in practice, especially when high power amplification (HPA) is regarded. As one can imagine, the continuous time signal shows a heavily different PMR. In this paper, the PMR of the continuous time signal is analyzed and its reduction by methods of signal shaping, especially SLM is addressed.

As one result a simple method for calculating an approximated PMR of the continuous time signal is presented. Moreover we show that the PMR of the continuous time signal of CP- and UW-OFDM can efficiently be reduced by means of SLM. In addition to that it is shown that CP-OFDM in combination with 4-QAM exhibits a tremendously higher PMR compared to UW-OFDM.

This paper is organised as follows: In Section II, the principles of SLM are reviewed. Two different OFDM schemes are considered: Unique-Word OFDM (UW-OFDM) and Cyclic-Prefix OFDM (CP-OFDM) are addressed in Section III. Section IV deals with simple methods to estimate the extremes in nominator and denominator in eq. (1) by means of digital signal processing. In Section V results are presented and a summary concludes the paper.

Notation: Lower-case bold face variables $(\mathbf{a}, \mathbf{b}, \dots)$ indicate vectors, and upper-case bold face variables $(\mathbf{A}, \mathbf{B}, \dots)$ indicate matrices. $[\mathbf{a}]_i$ is used to access the element i of the vector $\mathbf{a}$ (starting at 0). With $[\mathbf{A}]_{k,l}$ the matrix element in row

k and column l is obtained. To distinguish between time and frequency domain variables, we use a tilde to express frequency domain vectors ($\tilde{\mathbf{a}}, \ldots$). We further use $\mathbb{R}$ to denote the set of real and $\mathbb{C}$ the set of complex numbers with j being the imaginary unit, $\Im\{c\}$ the imaginary and $\Re\{c\}$ the real part of complex number c. $\mathbf{I}$ denotes the identity matrix, $\mathbf{0}$ an all-zero vector and $(\cdot)^T$ transposition. For all signals and systems the usual equivalent complex baseband representation (ECB) is applied.

II. Review of SLM

Here, we briefly review the basic principles of SLM (cf. [5]). The idea is simple:

1) Generate V different signal candidates via different bijective mappings of data to signal
2) Select the best signal candidate with respect to a certain feature, which is intended to be optimized

This process is visualized in the block diagram in Fig. 2 for the use in OFDM transmitters, where $\tilde{\mathbf{d}}$ represents the original data, the mappings $\mathcal{M}_i, i = 1 \ldots V$, create the V signal candidates $\tilde{\mathbf{d}}_i, i = 1 \ldots V$. The OFDM modulation leads to the time domain signal candidates $\mathbf{x}_i, i = 1 \ldots V$, amongst which the best candidate is chosen. The candidate selection usually has to be transmitted as side information, but there are methods without need of an explicit transmission of side information as well [6]. The receiver performs the inverse mapping to access the original data. Numerous methods for implementing such mappings exist in literature: quasi-random phase rotation [5], interleaving [7], scrambling [6], PTS [8], to name only a few.

Let the feature of the transmit signal which is intended to be optimized, be expressed by a random variable X which should be minimized. The behaviour of the signal w.r.t. this feature is usually characterized by the complementary cumulative distribution function (CCDF) of this variable X: $\mathrm{ccdf}_X(x) = \Pr(X > x)$. Applying SLM with mappings that guarantee quasi-independent signal candidates the $\mathrm{ccdf}_X^{\mathrm{SLM}}(x)$ for the best out of V candidates reads: $\mathrm{ccdf}_X^{\mathrm{SLM}}(x) = (\mathrm{ccdf}_X(x))^V$.

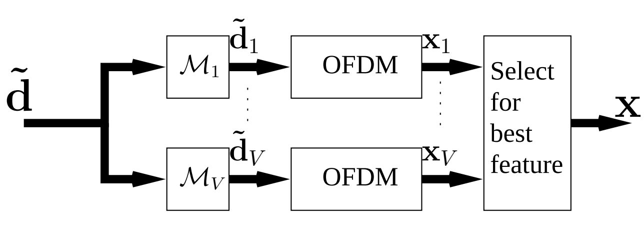

Fig. 2. Block diagramm: Selected Mapping for OFDM

III. OFDM: Cyclic Prefix and Unique Word

The main difference between UW-OFDM and CP-OFDM lies in the implementation of the guard interval. Whereas the cyclic prefix is data-dependent, the unique word is a deterministic sequence. Also, the unique word is part of the IDFT-interval in contrast to the cyclic prefix. Therefore the signal generation is different for both OFDM modulation schemes.

First, we want to look at the generation of CP-OFDM transmit signals. The data to be transmitted are contained in a frequency domain vector $\tilde{\mathbf{d}} \in \mathbb{C}^{N_d}$. The elements of this vector are taken from the QAM alphabets. At first, the IDFT generates the vector $\mathbf{y} = \mathbf{F}_N^{-1}\mathbf{B}\tilde{\mathbf{d}}$ with DFT-Matrix $[\mathbf{F}_N]_{k,l} = \mathrm{e}^{-\mathrm{j}\frac{2\pi}{N}kl}$ for $k, l = 0 \ldots N-1$ and a matrix $\mathbf{B}$ to formally introduce zero subcarriers. The CP is generated by repeating the last N_g elements of $\mathbf{y}$:

$$\mathbf{x}^{\mathrm{CP}} = \begin{bmatrix} [\mathbf{y}]_{N_d - N_g} \ldots [\mathbf{y}]_{N_d - 1} & \mathbf{y} \end{bmatrix}^{\mathrm{T}} \tag{2}$$

Now, we briefly review the generation of UW-OFDM transmit signals as presented previously (e.g. [9]). At first, an all-zero word is produced as a placeholder: $\mathbf{x}^{\mathrm{UW}} = \begin{bmatrix} \mathbf{x}_d^T & \mathbf{0}^T \end{bmatrix}^T$. Then, the unique-word $\mathbf{x}_u$ is added in time domain which leads to the transmit vector $\mathbf{x}' = \mathbf{x}^{\mathrm{UW}} + \begin{bmatrix} \mathbf{0}^T & \mathbf{x}_u^T \end{bmatrix}^T$. The time domain vector $\mathbf{x}$ is the IDFT of $\tilde{\mathbf{x}}$. $\tilde{\mathbf{x}}$ again consists of data subcarriers and zero subcarriers but additionally of redundant subcarriers $\tilde{\mathbf{r}} \in \mathbb{C}^{N_g}$. The zero subcarriers are generated by the matrix $\mathbf{B}$. Furthermore, a matrix $\mathbf{P}$ is introduced to permute the redundant subcarriers[1]. This leads to the following equation:

$$\mathbf{F}_N^{-1}\mathbf{B}\mathbf{P}\begin{bmatrix} \tilde{\mathbf{d}} \\ \tilde{\mathbf{r}} \end{bmatrix} = \begin{bmatrix} \mathbf{x}_d \\ \mathbf{0} \end{bmatrix}. \tag{3}$$

Rephrasing eq. (3) as $\left[\begin{smallmatrix} \mathbf{M}_{11} & \mathbf{M}_{12} \\ \mathbf{M}_{21} & \mathbf{M}_{22} \end{smallmatrix}\right]\left[\begin{smallmatrix} \tilde{\mathbf{d}} \\ \tilde{\mathbf{r}} \end{smallmatrix}\right] = \left[\begin{smallmatrix} \mathbf{x}_d \\ \mathbf{0} \end{smallmatrix}\right]$ and using appropriate sized submatrices $\mathbf{M}_{21}$ and $\mathbf{M}_{22}$, one can derive the relationship between the data subcarriers $\tilde{\mathbf{d}}$ and the redundant subcarriers $\tilde{\mathbf{r}}$ from $\mathbf{M}_{21}\tilde{\mathbf{d}} + \mathbf{M}_{22}\tilde{\mathbf{r}} = \mathbf{0}$. Consequently the redundant subcarriers are given by:

$$\tilde{\mathbf{r}} = -\mathbf{M}_{22}^{-1}\mathbf{M}_{21}\tilde{\mathbf{d}} = \mathbf{T}\tilde{\mathbf{d}} \quad , \quad \mathbf{T} \in \mathbb{C}^{N_r \times N_d}. \tag{4}$$

The UW-OFDM transmit signal (without the unique word $\mathbf{x}_u$) is obtained by:

$$\mathbf{x}^{\mathrm{UW}} = \mathbf{F}_N^{-1}\mathbf{B}\mathbf{P}\begin{bmatrix} \mathbf{I} \\ \mathbf{T} \end{bmatrix}\tilde{\mathbf{d}}. \tag{5}$$

In our simulations (cf. Section V) we did not look at a certain unique word. Thus, PMR is analysed based on vectors $\mathbf{x}_d$ and $\mathbf{y}$ for UW- and CP-OFDM, respectively.

The simulation setup is closely related to the 802.11a,g Wifi-Standard [11]. The system parameters are given in Table I. For the simulations, we used 4-QAM as modulation scheme for the data subcarriers.

TABLE I
System setting: CP-OFDM and UW-OFDM

	802.11a (CP-OFDM)	UW-OFDM
DFT length (N)	64	64
Used subcarriers per block	52	52
Data subcarriers	52	36
Length of guard interval (N_g)	16	16

[1] In previews works it has been shown that the average power in redundant subcarriers is lower for nearly equidistant redundant subcarriers [10]

Zero subcarriers are necessary at the DC subcarrier and the band edges, hence at the indices $\{0, 27, 28, ..., 37\}$. In [10], the ideal positions for the redundant subcarriers have been calculated to be at $\{2, 6, 10, 14, 17, 21, 24, 26, 38, 40, 43, 47, 50, 54, 58, 62\}$.

IV. Extremes of the continuous time signal

In [4], we derived an analytical expression for the CCDF of the PMR for statistical independent samples at symbol rate. Of course, this provides only a bound: the PMR of continuous signals has to be bigger than the one at symbol rate. So to reduce the actual PMR by means of signal processing within the transmitter, the continuous time signal has to be approximated in some way. The usual approach to approximate the continuous time signal is oversampling. Due to the zero subcarriers at band edges, this result for the oversampled representation is valid for any continuous time interpolation with a pulse which has a constant spectrum in the regime of nonzero subcarriers, e.g. for square-root Nyquist pulses with appropriate roll-off. To get an oversampled version $\mathbf{x}_{\mathrm{os}}$ of the signal (oversampling factor L), the elements of the signal vector are calculated by:

$$[\mathbf{x}_{\mathrm{os}}]_k = \sum_{i=0}^{N/2-1} [\tilde{\mathbf{x}}]_i \mathrm{e}^{\mathrm{j}\frac{2\pi}{LN}ki} + \sum_{i=N/2}^{N-1} [\tilde{\mathbf{x}}]_i \mathrm{e}^{\mathrm{j}\frac{2\pi}{LN}k(i-N+LN)}, \quad k = 0 \ldots (LN-1) \tag{6}$$

The vector $\tilde{\mathbf{x}}$ in eq. (6) is given by $\mathbf{F}_N \mathbf{x}^{\mathrm{UW}}$ for UW-OFDM and by $\mathbf{F}_N \mathbf{y}$ for CP-OFDM. To approximate maxima of the continuous time signal $\mathrm{s}(t)$, one could further use spline interpolation to get a higher resolution. Given a sufficient oversampling factor, the maxima calculated by spline interpolation proofed to be just marginally bigger than the ones given by the oversampled signal. So for determination of the maxima of the signal, we can simply use the oversampled signal $\mathbf{x}_{\mathrm{os}}$.

The situation is different for calculation of signal minima: As a minimum may lie anywhere between two signal points, a very high oversampling rate would be necessary to get a realistic impression of the PMR of the continuous time signal. This of course, is not feasible. On the other hand, given a reasonable oversampling factor the signal can be interpolated appropriately by straight lines in the complex plane between two signal points $x_i \in \mathbb{C}$ and $x_{i+1} \in \mathbb{C}$, $i = 0, \ldots, (LN-2)$, from the oversampled signal vector $\mathbf{x}_{\mathrm{os}}$. This straight line might produce a new minimum distance to the origin point at the point $l_i \in \mathbb{C}$, i.e. an approximation of the minimum of the continuous time signal, as shown in Fig. 3. To calculate this possible minimum, we introduce $a_i = x_{i+1} - x_i$ that describes the points on the straight line from x_i to x_{i+1} by $x_i + \alpha \cdot a_i$ with $\alpha \in [0, 1] \in \mathbb{R}$. The minimum is given by $l_i = \beta a'_i$ with $a'_i = \Im\{a_i\} - \mathrm{j}\Re\{a_i\}$ being a point orthogonal to a_i and $\beta \in \mathbb{R}$ a scaling factor. Consequently, the parameters α and β have to be determined for the point of intersection which leads to the following equation:

$$\beta \cdot a'_i = x_i + \alpha \cdot a_i \tag{7}$$

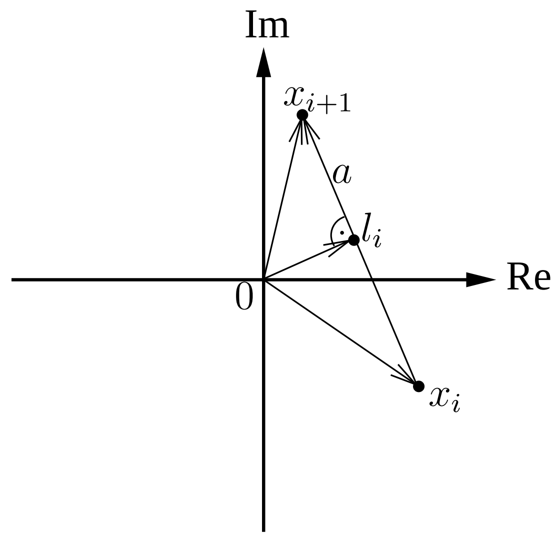

Fig. 3. Complex plane: Point with minimum magnitude between two signal samples

The solution for α and β is straight forward and leads to:

$$\alpha = -\frac{\Re\{x_i\}\Re\{a_i\} + \Im\{x_i\}\Im\{a_i\}}{|a_i|^2} \tag{8}$$

$$\beta = \frac{\Re\{x_i\}\Im\{a_i\} - \Im\{x_i\}\Re\{a_i\}}{|a_i|^2} \tag{9}$$

The point with minimum distance to the origin on the straight line described by x_i and x_{i+1} is given by $l_i = \beta \cdot a'_i$. Noteworthy, only for $\alpha \in [0, 1]$ this point lies between the points x_i and x_{i+1} and gives a valid minimum. To calculate the minima, we define the following mapping:

$$[\mathbf{l}]_i = \begin{cases} |\beta \cdot a'_i|, & \text{for } \alpha \in [0,1] \\ \min(|x_i|, |x_{i+1}|), & \text{otherwise} \end{cases} \quad i = 0, \ldots, (LN-2), \tag{10}$$

The vector $\mathbf{l}$ contains well approximated minima for the continuous time signal. Given this vector, we can define an approximated peak-to-minimum power ratio for one OFDM frame:

$$\mathrm{PMR}_{\mathrm{approx}} = \frac{\max_i |[\mathbf{x}_{\mathrm{os}}]_i|^2}{\min_j |[\mathbf{l}]_j|^2} \tag{11}$$

V. Results

In this section, we present simulation results to demonstrate the effect of SLM. We applied SLM to UW-OFDM and CP-OFDM with the simulation parameters given in Section III. Before using the interpolation method from eq. (10) to get an approximated PMR according to eq. (11), we oversampled the signal with an oversampling factor of $L = 8$ in eq. (6). Fig. 4 shows CCDF's of PMR without and with SLM with $V = 8$ different representations for the T-spaced discrete time and the continuous time transmit signal in case of UW-OFDM. As mentioned in [4], the CCDF of the discrete time signal very closely meets the analytical approach. For the continuous time signal, PMR is much higher due to possible smaller minima between samples even in a highly oversampled representation. But SLM proofs to be a very powerful tool to avoid extreme high values of PMR.

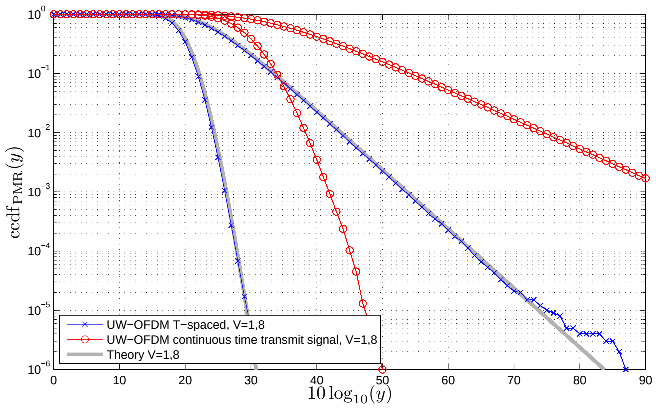

Fig. 4. CCDF curves for discrete time and continuous time transmit signals for $V = 1$ and $V = 8$ signal candidates

In Fig. 5, CCDF's of PMR of the continuous time transmit signal for UW- and CP-OFDM for various numbers V of alternatives of signals are presented. In contrast to UW-OFDM, the curve for CP-OFDM flattens at a certain probability. This behaviour is a result of the restriction of symbols to the 4-QAM alphabet: With a certain probability, the IDFT generates a time domain sample that equals exactly zero. This leads to a PMR of infinity and results in the flattening curve already for the signal sampled at symbol rate. In contrast, for UW-OFDM the redundant subcarriers $\tilde{\mathbf{r}}$ are not restricted to an integer grid. By this, the probability of zero components in vector $\mathbf{x}^{\mathrm{UW}}$ is much lower than in vector $\mathbf{x}^{\mathrm{CP}}$. Thus flattening is completely avoided which is a further feature of benefit of UW-OFDM over CP-OFDM.

SLM reduces the PMR significantly. For an excession probability of $\Pr(\mathrm{PMR} > x) = 10^{-3}$, SLM achieves a gain of 55 dB for $V = 16$ signal candidates for UW-OFDM. For CP-OFDM, the gain tends to infinity. Furthermore, it can be seen that the average PMR for UW-OFDM even with $V = 16$ signal candidates is lower than the average PMR of CP-OFDM.

To illustrate this fact, snapshots of signal constellations in the complex plane with and without SLM at symbol rate are given in Fig. 6. The signal constellations without SLM are depicted in blue crosses, the signal points after applying SLM are shown as red circles. Moreover the minima and maxima for the constellation with/without SLM are highlighted by a solid/dash-dotted line respectively. For CP-OFDM, it can be seen that SLM increases both the minimum and the maximum. The unwanted effect of increasing the maximum is caused by constellations with time domain samples equal to zero. Due to this reason, SLM has much fewer "good" alternatives with high minimum value and low maximum value to be chosen from. UW-OFDM reveals a different behaviour: the redundant subcarriers are not on the regular 4-QAM grid and no zero samples in time domain are generated. Therefore it is more likely that SLM finds a candidate with increased minimum and decreased peak value (as shown in Fig. 6).

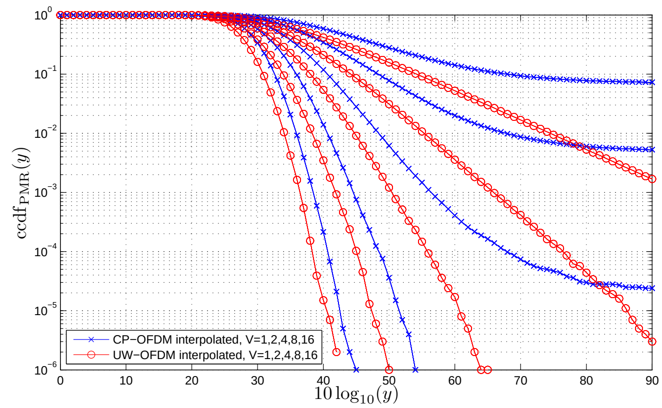

Fig. 5. CCDF curves for continuous time CP- and UW-OFDM transmit signals

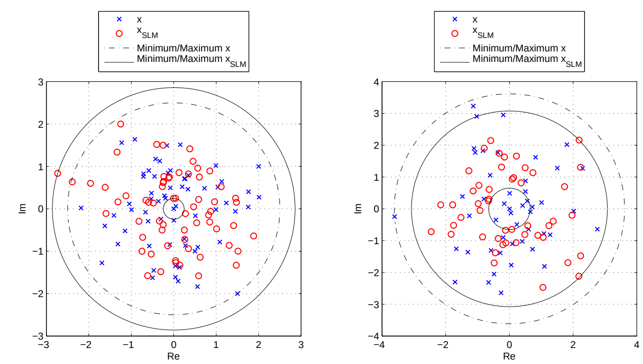

Fig. 6. Snapshot: CP-OFDM (left) and UW-OFDM (right) with and without SLM

VI. Conclusion

In this paper we addressed the issue of optimizing CP-OFDM and UW-OFDM signals for polar amplifier structures which require a low PMR of the signal to be amplified. For calculation of the PMR of the analog transmit signal an interpolation method is derived. SLM is used to reduce PMR of this interpolated signal. By means of simulation results it is shown, that the PMR can be reduced by SLM by a very huge amount and that in general UW-OFDM has a smaller PMR than CP-OFDM.

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