Soft-Output Sphere Detection for Coded Unique Word OFDM

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***Abstract*—Unique Word (UW) OFDM is an attractive alternative to conventional cyclic-prefix OFDM as it offers the potential to apply more sophisticated detection schemes, resulting in improved performance both in uncoded as well as coded transmission. In this paper, we study soft-output detection schemes suited for coded UW-OFDM transmission. In particular, we present an efficient implementation of near-optimum soft-output detection based on the soft-output sphere decoder (SOSD) algorithm. The performance and complexity of this approach is compared to a low-complexity linear detector (linear minimum mean squared error, LMMSE), which uses exact statistics of the residual noise for computation of the soft output. The conducted numerical simulations emphasize that the SOSD constitutes the decoding reference for uncoded as well as coded UW-OFDM transmission. Nevertheless, the LMMSE offers a good performance trade-off at a constant complexity, while the SD's complexity is variable.**

I. Introduction

Unique word (UW) OFDM uses a deterministic sequence in the guard interval of an OFDM symbol, instead of the cyclic prefix used in most conventional OFDM systems [1]. By putting the UW inside the DFT (discrete Fourier transform) interval in time domain, which is achieved by loading certain subcarriers in frequency domain with redundant symbols instead of data, the intended cyclicity of the OFDM symbol is ensured. As the redundant subcarriers depend on the data, the introduced correlations can be exploited by the receiver, enabling much more sophisticated data estimation strategies compared to those applicable for conventional OFDM.

In this paper we focus on soft-output detection schemes for UW-OFDM in the presence of channel coding. The system model of UW-OFDM can be interpreted as a MIMO (multiple-input multiple-output) channel, thus, in principle, all the receivers known for MIMO detection can be used with single-antenna UW-OFDM systems. The best known receiver for this channel type is executing a maximum-likelihood sequence estimation (MLSE), which can efficiently be implemented using the Sphere Decoder (SD) algorithm [2], [3]. In this paper, we extend this approach to further include the computation of reliability information on the detected symbols, i.e. so-called soft-output MLSE. Following the approach for MIMO detection [4], we present an efficient implementation based on the single tree search soft-output SD. However, as the computational complexity of soft-output MLSE varies with the channel conditions and may even be prohibitively high, we compare it to a second soft-output detector given by the LMMSE (linear minimum mean square error) data estimator [1], which has lower and in particular constant complexity.

The paper is organized as follows: After the definition of the UW-OFDM system model in Sec. II, soft-output MLSE and its implementation using the SD is presented in Sec. III. Following a brief review of LMMSE data estimation in Sec. IV, we compare the performance of the respective schemes by means of numerical simulations in Sec. V and conclude with a summary in Sec. VI.

II. Unique Word OFDM System Model

We briefly review the approach of UW-OFDM first in its systematic variant (see [1] for further details) and then introduce modifications for a non-systematic approach [5].

A. Systematic UW-OFDM Symbol Generation

Let $\mathbf{x}_u$ be a predefined sequence of length N_u, the so-called unique word, forming the tail of the UW-OFDM symbol. Hence, the time domain symbol vector consists of two parts $\begin{bmatrix}\mathbf{x}_d^\mathsf{T} & \mathbf{x}_u^\mathsf{T}\end{bmatrix}^\mathsf{T} \in \mathbb{C}^{N\times 1}$, where $\mathbf{x}_d \in \mathbb{C}^{(N-N_u)\times 1}$ is the information-bearing part affected by the data symbols. It is advantageous to generate an OFDM symbol $\mathbf{x} = \begin{bmatrix}\mathbf{x}_d^\mathsf{T} & \mathbf{0}^\mathsf{T}\end{bmatrix}^\mathsf{T}$ with a zero UW in a first step, as the result of a length N IDFT (inverse DFT), and to add the desired UW to determine the transmit symbol $\mathbf{x}' = \mathbf{x} + \begin{bmatrix}\mathbf{0}^\mathsf{T} & \mathbf{x}_u^\mathsf{T}\end{bmatrix}^\mathsf{T}$ in a second step [6]. As we intend N_u zero samples as a part of the IDFT output, at least the same number of subcarriers in frequency domain must be set to appropriate values. We spend $N_r = N_u$ subcarriers and name them *redundant* subcarriers, as they are loaded with values depending on the data vector $\tilde{\mathbf{d}} \in \mathbb{C}^{N_d\times 1}$.

Following the derivations in [1] and employing the N point DFT matrix $\mathbf{F}_N$ with its element in the k-th row and the l-th column $[\mathbf{F}_N]_{kl} = \mathrm{e}^{-\mathrm{j}\frac{2\pi}{N}kl}$, the transmit symbol can be written as $\mathbf{x}' = \mathbf{F}_N^{-1}\mathbf{G}\tilde{\mathbf{d}} + \begin{bmatrix}\mathbf{0}^\mathsf{T} & \mathbf{x}_u^\mathsf{T}\end{bmatrix}^\mathsf{T}$, utilizing the UW-OFDM symbol generator matrix $\mathbf{G} \in \mathbb{C}^{N\times N_d}$, which determines the values on the redundant subcarriers in order to achieve a zero word in time domain. The generator matrix

$$\mathbf{G} = \mathbf{P}\begin{bmatrix}\mathbf{I} \\ \mathbf{T}\end{bmatrix} \tag{1}$$

consists of a matrix $\mathbf{T} \in \mathbb{C}^{N_r\times N_d}$ to determine the values on the redundant subcarriers, which is calculated in order to

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satisfy the zero word constraint at the output of the IDFT. The permutation matrix $\mathbf{P} \in \{0,1\}^{N \times N}$ moves the data and redundant values to their dedicated subcarriers and results from an optimization, that aims to minimize the mean energy on the redundant subcarriers. Both are detailed in [1].

Transforming the UW into frequency domain $\tilde{\mathbf{x}}_u = \mathbf{F}_N \begin{bmatrix} \mathbf{0}^\mathsf{T} & \mathbf{x}_u^\mathsf{T} \end{bmatrix}^\mathsf{T}$ allows us to rewrite this relationship as

$$\mathbf{x}' = \mathbf{F}_N^{-1}(\mathbf{G}\tilde{\mathbf{d}} + \tilde{\mathbf{x}}_u). \tag{2}$$

The propagation of the OFDM symbol, assembled according to (2), through a multi-path channel is modeled using a cyclic convolution matrix $\mathbf{H}_c$ and a noise vector $\mathbf{n}$ with zero mean and the covariance matrix $\mathbf{C}_{nn} = \sigma_n^2 \mathbf{I}$. After applying the DFT at the receiver, the frequency domain receive vector can be formulated as

$$\tilde{\mathbf{y}}_r = \mathbf{F}_N \mathbf{H}_c \mathbf{x}' + \mathbf{F}_N \mathbf{n} = \mathbf{F}_N \mathbf{H}_c \mathbf{F}_N^{-1}(\mathbf{G}\tilde{\mathbf{d}} + \tilde{\mathbf{x}}_u) + \tilde{\mathbf{v}}, \tag{3}$$

where $\tilde{\mathbf{v}} = \mathbf{F}_N \mathbf{n}$. The matrix $\widetilde{\mathbf{H}} = \mathbf{F}_N \mathbf{H}_c \mathbf{F}_N^{-1}$ is diagonal and contains the sampled channel frequency response on its main diagonal. As a first receiver processing step we subtract the UW influence to obtain the symbol $\tilde{\mathbf{y}} = \tilde{\mathbf{y}}_r - \widetilde{\mathbf{H}}\tilde{\mathbf{x}}_u$

$$\tilde{\mathbf{y}} = \widetilde{\mathbf{H}}\mathbf{G}\tilde{\mathbf{d}} + \tilde{\mathbf{v}}. \tag{4}$$

At this point, the channel propagation matrix $\widetilde{\mathbf{H}}\mathbf{G} \in \mathbb{C}^{N \times N_d}$ can be interpreted as the channel matrix of a complex MIMO channel, although we only consider a single antenna system. This allows us to treat the UW-OFDM system as a MIMO system with N_d input and $N_d + N_r$ output streams. All MIMO detection methods can be used to recover the data. Furthermore, due to the definition of zeros at some IDFT output positions, the UW-OFDM symbol generation model can be considered as a complex valued RS-code.

B. Non-Systematic UW-OFDM Symbol Generation

In the last section we showed the basic UW-OFDM symbol structure, where each subcarrier is dedicated to carry either data or redundant values. In [5] a *non-systematic* generation of UW-OFDM symbols is suggested, that lifts off the dedication of the subcarriers and improves the system performance. More precisely, an altered generator matrix $\breve{\mathbf{G}}$ is employed, that distributes the redundancy over all subcarriers in use. As the data is not immediately visible in the UW-OFDM symbol anymore, this approach is called *non-systematic* generation, analogously to the same term in channel coding. The other way round, *systematically* generated UW-OFDM symbols possess dedicated carriers, that directly show the information symbols.

The goal in [5] was to find a generator matrix

$$\breve{\mathbf{G}} = \mathbf{A}\mathbf{P}\begin{bmatrix} \mathbf{I} \\ \breve{\mathbf{T}} \end{bmatrix}, \tag{5}$$

with a yet-to-find real matrix $\mathbf{A} \in \mathbb{R}^{N \times N}$, that minimizes the sum of the error variances after LMMSE data estimation for the AWGN (additive white Gaussian noise) channel case, and at a fixed $c = E_s/\sigma_n^2$, where E_s denotes the mean energy per data symbol, cf. [5, Eq. (36)]. The cost function for the minimization is based on the covariance matrix of the estimation error $\mathbf{C}_{ee}$, as defined later in (13), and can be treated as a function of the non-singular matrix $\mathbf{A}$, that distributes the redundancy over all subcarriers. However, the solution to this optimization problem is ambiguous. Particular solutions can be found, e.g. by applying the steepest descent algorithm. As in [5, Eq. (44)] we choose the initialization $\mathbf{A}^{(0)} = \mathbf{I}$, which implies $\breve{\mathbf{T}}^{(0)} = \mathbf{T}$ and $\breve{\mathbf{G}}^{(0)} = \mathbf{G}$. The iterative optimization process consequently starts with the code generator matrix $\mathbf{G}$ of the systematic UW-OFDM concept, which can be considered a good initial guess. Note, that again $\breve{\mathbf{T}}$ is unambiguously given by the zero word constraint after the IDFT.

III. Soft-Output Sphere Decoding

For the usual case of equi-probable data sequences, the best decoding results in the recovery of $\tilde{\mathbf{d}}$ from (4) are achieved by maximum-likelihood sequence estimation (MLSE) on each OFDM symbol. Since the receive signal is corrupted by AWGN, using the above linear system model (4), the MLSE problem translates to the minimization of the distance of all possible OFDM symbols after channel propagation to the received vector:

$$\tilde{\mathbf{d}}^{\mathrm{ML}} = \underset{\tilde{\mathbf{d}} \in \mathcal{A}^{N_d}}{\operatorname{argmin}} \, ||\widetilde{\mathbf{H}}\mathbf{G}\tilde{\mathbf{d}} - \tilde{\mathbf{y}}||_2^2 \tag{6}$$

Every vector $\tilde{\mathbf{d}} \in \mathcal{A}^{(N_d \times 1)}$ of symbols from the alphabet $\mathcal{A}$ can also be represented by its binary equivalent $\mathbf{b} \in \{0,1\}^{(N_d B \times 1)}$, when B bits form each symbol from $\mathcal{A}$. In coded UW-OFDM the end-to-end performance is improved by delivering reliability information on every detected bit b_l, i.e. soft output, to the subsequent channel decoder, e.g. a soft-input Viterbi decoder in case of convolutional coding.

As soft information we use log-likelihood ratios (LLRs) on the code bits $\mathbf{b}^{\mathrm{ML}}$ representing the estimated MLSE data vector $\tilde{\mathbf{d}}^{\mathrm{ML}}$. Due to the large dimensionality of the search problem, we employ the max-log approximation. Assuming equi-probable source bits, the LLRs can be written as [4]

$$L_l = \begin{cases} \lambda^{\mathrm{ML}} - \lambda_l^{\overline{\mathrm{ML}}} & \text{if } b_l^{\mathrm{ML}} = 0, \\ \lambda_l^{\overline{\mathrm{ML}}} - \lambda^{\mathrm{ML}} & \text{if } b_l^{\mathrm{ML}} = 1, \end{cases} \tag{7}$$

with the bit number $l = 0, \ldots, N_d B - 1$, the metric of the ML solution $\lambda^{\mathrm{ML}} = \left\| \widetilde{\mathbf{H}}\mathbf{G}\hat{\tilde{\mathbf{d}}}^{\mathrm{ML}} - \tilde{\mathbf{y}} \right\|^2$ introduced in (6), and the metric of the counter-hypothesis

$$\lambda_l^{\overline{\mathrm{ML}}} = \min_{\tilde{\mathbf{d}} \in \mathcal{A}_l^{\left(\overline{b_l^{\mathrm{ML}}}\right)}} \left\| \widetilde{\mathbf{H}}\mathbf{G}\tilde{\mathbf{d}} - \tilde{\mathbf{y}} \right\|^2. \tag{8}$$

The metric of the counter-hypothesis is specific to each bit and determined by fixing the l-th bit of the data vector $\tilde{\mathbf{d}}$ under test to the complement of the MLSE solution $\mathbf{b}^{\mathrm{ML}}$. The flipped bit is denoted as the counter-hypothesis $\overline{b_l^{\mathrm{ML}}}$ and $\mathcal{A}_l^{(\beta)}$ is the set of all valid data vectors $\tilde{\mathbf{d}}$ with the l-th bit equal to β.

In brute-force MLSE *every* possible data vector $\tilde{\mathbf{d}}$ needs to be considered, which is an almost impossible task, even with the slimmed down system as given in Sec. V-A. For this kind of problem the Sphere Decoder algorithm is an attractive method, as it is able to solve (6) in a tractable amount of time. An adapted version of the algorithm in [2], that initially does not provide soft output, was implemented for Unique Word OFDM in [3].

To allow for Sphere Decoding, a QR decomposition of the transmission matrix

$$\widetilde{\mathbf{H}}\mathbf{G} = \begin{bmatrix}\mathbf{Q}_1 & \mathbf{Q}_2\end{bmatrix}\begin{bmatrix}\mathbf{R}\\ \mathbf{0}\end{bmatrix} \tag{9}$$

enables the required simplifications, where $\begin{bmatrix}\mathbf{Q}_1 & \mathbf{Q}_2\end{bmatrix} = \mathbf{Q} \in \mathbb{C}^{(N\times N)}$ is a unitary matrix and $\mathbf{R} \in \mathbb{C}^{N_d\times N_d}$ is upper triangular.

Thus, the term in (6) to be minimized becomes [3]

$$\begin{aligned}\left\|\widetilde{\mathbf{H}}\mathbf{G}\tilde{\mathbf{d}} - \tilde{\mathbf{y}}\right\|_2^2 &= \left\|\mathbf{Q}\begin{bmatrix}\mathbf{R}\\ \mathbf{0}\end{bmatrix}\tilde{\mathbf{d}} - \tilde{\mathbf{y}}\right\|_2^2 \\ &= \left\|\begin{bmatrix}\mathbf{R}\\ \mathbf{0}\end{bmatrix}\tilde{\mathbf{d}} - \begin{bmatrix}\mathbf{Q}_1^{\mathsf{H}}\\ \mathbf{Q}_2^{\mathsf{H}}\end{bmatrix}\tilde{\mathbf{y}}\right\|_2^2 \\ &= \left\|\mathbf{R}\tilde{\mathbf{d}} - \mathbf{Q}_1^{\mathsf{H}}\tilde{\mathbf{y}}\right\|_2^2 + \left\|\mathbf{Q}_2^{\mathsf{H}}\tilde{\mathbf{y}}\right\|_2^2. \end{aligned} \tag{10}$$

As the second term is independent of $\tilde{\mathbf{d}}$, the minimization problem (6) transforms into

$$\tilde{\mathbf{d}}^{\mathrm{ML}} = \underset{\tilde{\mathbf{d}}\in\mathcal{A}^{N_d}}{\operatorname{argmin}} \left\|\mathbf{R}\tilde{\mathbf{d}} - \tilde{\mathbf{y}}'\right\|_2^2, \tag{11}$$

defining $\tilde{\mathbf{y}}' = \mathbf{Q}_1^{\mathsf{H}}\tilde{\mathbf{y}}$. Due to the triangular structure of $\mathbf{R}$, (11) can be solved in a recursive fashion using the Sphere Decoder algorithm in [2].

Calculating the LLRs resorts to finding the minimum of an unrestricted tree search, i.e. (6), and the N_dB "next-best" minima of (8) for each bit. One could solve these minimization problems subsequently by re-running the SD for each counter-hypothesis with correspondingly restricted search space, i.e. perform a so-called repeated tree search. This however requires to run the SD N_dB+1 times per OFDM symbol, and hence, imposes a high complexity burden.

To alleviate this, we extend the initial algorithm published in [2], [3] to provide soft information according to the *single tree search* principle shown in [4], [7], which ensures that one leaf of the SD tree is visited not more than once and branches are only followed if there is the chance to update either $\tilde{\mathbf{d}}^{\mathrm{ML}}$ (and thus λ^{ML}) or one of the N_dB counter-hypotheses' metrics $\lambda_l^{\overline{\mathrm{ML}}}$. We implemented this single tree search and introduced a restriction to a finite, possibly complex valued transmit symbol alphabet $\mathcal{A}$, instead of the ability to operate on an infinite real lattice only. Our implementation is outlined in Alg. 1, following the pseudo-code notation in [2], [8], and is available in [9].

The function INITLIST$(k, e_k^{(k)}, \left[\mathbf{R}^{-1}\right]_{kk}, \mathcal{A})$ creates a list of all symbols of the alphabet $\mathcal{A}$, that is sorted by the distance $\left|e_k^{(k)} - \tilde{d}_k\right|^2$ of the received symbol on level k to the possible undisturbed symbol $\tilde{d}_k$ in ascending order, together with the corresponding normalized distance $\delta = \left(e_k^{(k)} - \tilde{d}_k\right)\left[\mathbf{R}^{-1}\right]_{kk}$.

Algorithm 1 Soft-Output Sphere Decoder

function $[\mathbf{b}^{\mathrm{ML}}, \mathbf{L}] =$ SOFT-OUTPUT SD$(\tilde{\mathbf{y}}', \mathbf{R}^{-1}, \mathcal{A})$

1: $\lambda^{\mathrm{ML}} \leftarrow \infty;\ \Delta_{N_d-1} \leftarrow 0;\ k \leftarrow N_d - 1$
2: $\lambda_l^{\overline{\mathrm{ML}}} \leftarrow \infty,\quad$ for $l = 0, \ldots, N_dB - 1$
3: $\mathbf{e}^{(k)} \leftarrow \mathbf{R}^{-1}\tilde{\mathbf{y}}'$
4: INITLIST$(k, e_k^{(k)}, \left[\mathbf{R}^{-1}\right]_{kk}, \mathcal{A})$
5: **while** $k < N_d$ **do**
6: $\quad [\tilde{d}_k, \mathbf{b}, \delta] =$ GETNEXTSYMBOL(k)
7: $\quad$ **if** isempty$(\tilde{\mathrm{d}}_{\mathrm{k}})$ **then** $\quad\triangleright$ all symbols from $\mathcal{A}$ tried
8: $\quad\quad k \leftarrow k + 1$
9: $\quad$ **else**
10: $\quad\quad \Delta_{\text{new}} \leftarrow \Delta_k + |\delta|^2$
11: $\quad\quad \rho \leftarrow \max \lambda_l^{\overline{\mathrm{ML}}}\Big|_{l=0,\ldots,(k+1)B-1\ \vee\ b_l \neq b_l^{\mathrm{ML}}}$
12: $\quad\quad$ **if** $\Delta_{\text{new}} < \rho$ **then**
13: $\quad\quad\quad$ **if** $k > 0$ **then**
14: $\quad\quad\quad\quad e_l^{(k-1)} \leftarrow e_l^{(k)} - \delta\left[\mathbf{R}^{-1}\right]_{lk},\ l = 0, \ldots, k-1$
15: $\quad\quad\quad\quad k \leftarrow k - 1$
16: $\quad\quad\quad\quad \Delta_k \leftarrow \Delta_{\text{new}}$
17: $\quad\quad\quad\quad$ INITLIST$(k, e_k^{(k)}, \left[\mathbf{R}^{-1}\right]_{kk}, \mathcal{A})$
18: $\quad\quad\quad$ **else**
19: $\quad\quad\quad\quad$ **if** $\Delta_{\text{new}} < \lambda^{\mathrm{ML}}$ **then**
20: $\quad\quad\quad\quad\quad \lambda_l^{\overline{\mathrm{ML}}} \leftarrow \lambda^{\mathrm{ML}},\quad \forall l$ with $b_l \neq b_l^{\mathrm{ML}}$
21: $\quad\quad\quad\quad\quad \tilde{\mathbf{d}}^{\mathrm{ML}} \leftarrow \tilde{\mathbf{d}};\ \lambda^{\mathrm{ML}} \leftarrow \Delta_{\text{new}}$
22: $\quad\quad\quad\quad$ **else**
23: $\quad\quad\quad\quad\quad \lambda_l^{\overline{\mathrm{ML}}} \leftarrow \min\left\{\lambda_l^{\overline{\mathrm{ML}}}, \Delta_k\right\},\quad \forall l$ with $b_l \neq b_l^{\mathrm{ML}}$
24: $\quad\quad\quad\quad$ **end if**
25: $\quad\quad\quad\quad \lambda_l^{\overline{\mathrm{ML}}} \leftarrow \min\left\{\lambda_l^{\overline{\mathrm{ML}}}, \lambda^{\mathrm{ML}} + L_{\max}\right\}$
26: $\quad\quad\quad$ **end if**
27: $\quad\quad$ **else**
28: $\quad\quad\quad k \leftarrow k + 1$
29: $\quad\quad$ **end if**
30: $\quad$ **end if**
31: **end while**
32: $L_l \leftarrow (\lambda^{\mathrm{ML}} - \lambda_l^{\overline{\mathrm{ML}}}) \cdot (2b_l^{\mathrm{ML}} - 1),\quad \forall l = 0, \ldots, N_dB - 1$

end function

One list is held separately for each level k. The function GETNEXTSYMBOL(k) pops off the first transmit symbol $\tilde{d}_k$ from the list for level k with the smallest distance, as determined in INITLIST before, and the normalized difference δ. After this, the used symbol is deleted from the list. If the list is already empty upon the call of the function, an empty symbol $\tilde{d}$ is returned to indicate this. In line 7 this is checked and provided for, by moving up one level.

LLR clipping during the SD search is an effective means to speed up the search process and hence reduce its computational complexity [4] at the cost of bit error performance. This is achieved by limiting the SD search radius ρ (line 11) to include only counter-hypotheses within the LLR clipping level $L_{\max}$ (cf. line 25). As a thorough investigation of the trade-off between performance and complexity enabled by LLR clipping is beyond the scope of this paper, we only state that for our setup an LLR clipping level of $L_{\max} = 5$ represents a good compromise.

The choice of the QR decomposition algorithm (9) required in the preprocessing step of the SD has significant influence on the execution time of the Sphere Decoder, as a well sorted QR decomposition helps in finding the needed hypotheses early. We apply the post-sorting algorithm using the Householder reflectors for the decomposition, cf. [10].

IV. Linear MMSE Detector

We compare the performance of the soft-output Sphere Decoder with a linear equalization receiver according to the MMSE criterion [1]. The data estimates are determined by first processing the receive symbols with a linear filter $\tilde{\mathbf{d}}^{\mathrm{LMMSE}} = \mathbf{E}^{\mathrm{LMMSE}}\tilde{\mathbf{y}}$, where

$$\mathbf{E}^{\mathrm{LMMSE}} = \left(\mathbf{G}^{\mathsf{H}}\widetilde{\mathbf{H}}^{\mathsf{H}}\widetilde{\mathbf{H}}\mathbf{G} + \frac{N\sigma_n^2}{\sigma_d^2}\mathbf{I}\right)^{-1}\mathbf{G}^{\mathsf{H}}\widetilde{\mathbf{H}}^{\mathsf{H}} \qquad (12)$$

is the MMSE equalizer matrix. We note that equalization using LMMSE is enabled by the distinct system structure of UW-OFDM; it is not usefully applicable for conventional cyclic-prefix OFDM, where the simple equalizer inverting the channel frequency response $\widetilde{\mathbf{H}}^{-1}$ is already the optimum receiver. The LMMSE estimator can be extended to provide soft information additionally. To this end, the equalized symbols $\tilde{\mathbf{d}}^{\mathrm{LMMSE}}$ are fed to a conventional soft-decision QAM demapper. However, due to the LMMSE filter, the remaining noise values at the input of this demapper are in general correlated. The covariance matrix of the remaining additive Gaussian noise vector is given by

$$\mathbf{C}_{ee} = N\sigma_n^2\left(\mathbf{G}^{\mathsf{H}}\widetilde{\mathbf{H}}^{\mathsf{H}}\widetilde{\mathbf{H}}\mathbf{G} + \frac{N\sigma_n^2}{\sigma_d^2}\mathbf{I}\right)^{-1}. \qquad (13)$$

The varying noise variances, given by the main diagonal elements of the error covariance matrix $\mathbf{C}_{ee}$ are taken into account in the computation of the soft output. The correlations are neglected. This soft-output detection scheme based on an LMMSE equalizer represents a low-complexity alternative to soft-output MLSE.

V. Simulation Results

A. Simulation Setup

A block diagram of the system setup used in this work for simulation is shown in Fig. 1. Before QAM mapping, the binary input data is channel coded by the widely used rate $1/2$ convolutional code with generator polynomials (133, 171). Then, the UW-OFDM symbol is assembled in frequency domain, transformed and supplied with the UW, resulting in the time domain signal to be transmitted. After channel propagation and subtraction of the UW influence according to (II-A), the data estimation is performed.

Since we do not utilize the UW itself in the considerations of this paper, we used the zero word as UW for a length 24 DFT UW-OFDM system for all our simulations. All system parameters are summarized in Tab. I. The number of redundant subcarriers as well as the size of the QAM alphabet is reduced compared to former simulated UW-OFDM systems [3] in order to obtain a computational complexity for the soft-output Sphere Decoder (soft-output SD) in feasible orders of magnitude.

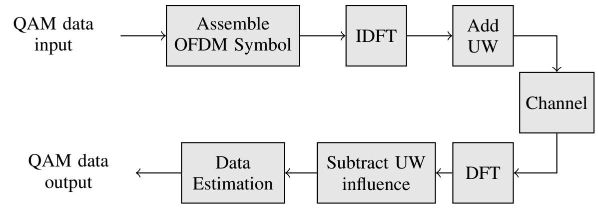

Fig. 1. Block diagram of the transceiver system used for simulation.

The soft-output data estimators are compared in terms of performance and complexity for a rate $1/2$ convolutional coded transmission. We consider an AWGN channel and, more importantly, a multi-path environment, averaging the results over a large number of channel realizations. Systematic as well as non-systematic generation of the UW-OFDM symbol, whose properties were discussed earlier, is studied. Code examples for the UW-OFDM setup are available in [9].

B. Bit Error Performance

Fig. 2 depicts the bit error ratio of a transmission over the AWGN channel for uncoded as well as coded transmission. In spite of the different parameter choices compared to [11], the uncoded results show the very same tendencies as in [11]. In the following we therefore only comment the coded results. In case of non-systematic generation of the UW-OFDM symbol, the performance of the soft-output SD and the LMMSE estimator widely coincides. Systematic generation of the UW-OFDM symbol induces a loss of approximately $0.5\,\mathrm{dB}$ at a BER of 10^{-5} for the soft-output SD, which is still about $0.3\,\mathrm{dB}$ superior to the LMMSE estimator. We note that due to the max-log approximation in the determination of the soft information the soft-output SD is not optimal, but still performs better than the LMMSE, which uses exact statistics of the residual noise as soft information.

The more relevant results for OFDM transmission are observed after transmission over a multi-path channel. Fig. 3 shows the results averaged over a set of 5 000 channels, which were generated according to the channel model defined for IEEE 802.11a [12] with a channel impulse response of length 9. All channel realizations feature (on average) an rms delay spread of $50\,\mathrm{ns}$ and all responses are normalized such that the receive power is independent of the actual

TABLE I
Parameters of the investigated UW-OFDM system

Modulation scheme		4-QAM
Coding rates		uncoded, 1/2
DFT length	N	24
No. of data (red.) subcarriers	N_d (N_r)	16 (8)
Indices of red. subcarriers		$\{1, 4, \ldots, 22\}$
Unique Word	$\mathbf{x}_u$	$\mathbf{0}^{(8\times 1)}$

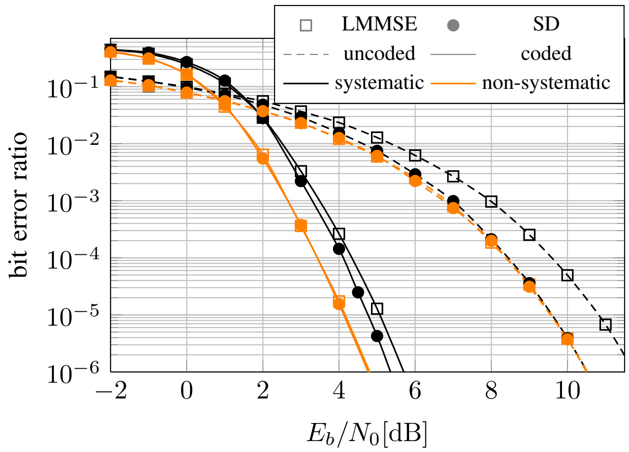

Fig. 2. Bit error performance results of the exemplary UW-OFDM system in the AWGN channel.

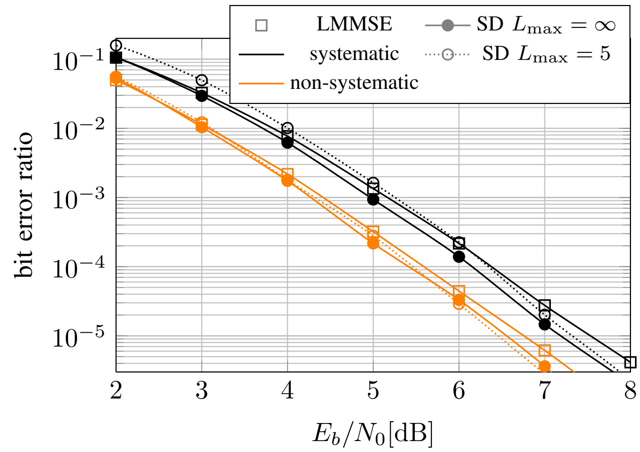

Fig. 4. Bit error performance results of the exemplary coded UW-OFDM system in multi-path environment for different LLR clipping levels $L_{\max}$.

channel. The uncoded results show a significant advantage of the SD compared to the LMMSE of 5.3 / 6.3 dB at a BER of 10^{-5}, using systematic / non-systematic UW-OFDM generation. As expected, the gap for the coded transmission is smaller but the soft-output SD still considerably outperforms the LMMSE. Especially for the non-systematic UW-OFDM symbol generation the remaining gain is substantial.

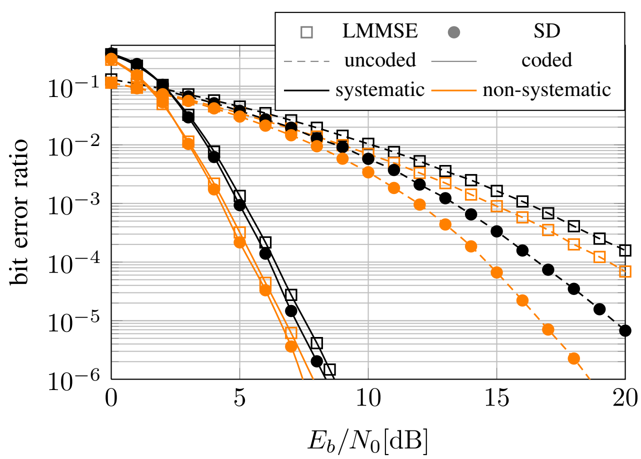

Fig. 3. Bit error performance results of the exemplary UW-OFDM system in multi-path environment.

In Fig. 4 the BER curves for coded transmission in multi-path environment with additional results for LLR clipping with level $L_{\max} = 5$ are enlarged. The LLR clipped detection shows some minor degradation, but still better results than the LMMSE at higher E_b/N_0 and thus poses a much less complex alternative to the full soft-output SD.

VI. Conclusion

In this paper, we have studied soft-output detection schemes for coded UW-OFDM. The distinct system structure of UW-OFDM enables to use detection schemes known for MIMO channels, even though considering only a single antenna system. We derived a near-optimum soft-output detection scheme and its implementation using the soft-output SD algorithm. The SD-based detector has been compared to data estimation using linear equalization, in particular LMMSE data estimation using the error covariance matrix as soft information. The LMMSE-based soft-output detector suffers some loss compared to the SD-based scheme, however it represents a competitive alternative due to its lower and in particular constant computational complexity.

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